Lecture 14: Integration by parts and trigonometric integrals

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Integration by parts

When we integrated by substitution previously, we were performing the reverse of the chain rule. By integrating by parts, we're reversing the product rule.

$$\frac{d}{dx}\big(f(x)g(x)\big) = g(x)f'(x) + g'(x)f(x))$$

integrate both sides to get rid of $\frac{d}{dx}$

$$\int f(x)g'(x)\,\mathrm{dx} = f(x)g(x) - \int f'(x)g(x)\,\mathrm{dx}$$

Examples:

$$\int x \cdot \sin x\,\mathrm{dx}$$

substitution doesnt work

$f(x)=x \quad g'(x)=\sin x$

$f'(x)=1 \quad g(x)=-\cos x$

$$= x(-\cos(x)) - \int 1(-1\cos(x))\,dx$$

$$= -x(\cos(x)) + \int \cos(x)\,dx$$

$$= -x(\cos(x)) + \sin(x) + C$$

$$\int x^2 e^x\,dx$$

$f(x)=x^2 \quad g'(x)=e^x$

$f'(x)=2x \quad g(x)=e^x$

$$= x^2e^x - \int 2xe^x\,dx$$

$$= x^2e^x - 2\int xe^x\,dx$$

integration by parts again

$f(x)=x \quad g'(x)=e^x$

$f'(x)=1 \quad g(x)=e^x$

$$= x^2e^x - 2(x\,e^x - \int 1e^x\,dx$$

$$= x^2e^x - 2xe^x - 2(-e^x)$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

$$\int \sin(x)\cos(x)\,dx$$

$f(x)=\sin(x) \quad g'(x)=\cos(x)$

$f'(x)=\cos(x) \quad g(x)=\sin(x)$

$$= (\sin(x))^2 - \int \sin(x)\cos(x)\,dx$$

$$= \sin^2 x - \frac{1}{2}\int \sin(2x)\,dx$$

substitution rule

$$= \sin^2 x - \frac{1}{2} * \frac{1}{2}(-\cos(2x)) + C$$

or use trig identity right away

use identity:

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\frac{1}{2}\sin(2x) = \sin(x)\cos(x)$$

NB

Half angle formulas:

$$\sin^2 x = \frac{1}{2}(1-\cos(2x))$$

$$\cos^2 x = \frac{1}{2}(1+\cos(2x))$$

$$= \sin^2 x - \frac{1}{2} * \frac{1}{2}(-\cos(2x)) + C$$

identity
right away

$$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$$

$\sin^2 x + \cos^2 x = 1$

$$\int \sin^3 x \, dx = \int \sin^2 x \sin(x) \, dx$$

substitution

$$= \int (1 - \cos^2 x)\sin(x) \, dx$$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = \frac{-du}{\sin x}$

$$= -\int (1 - u^2) \cancel{\sin(x)} \left(\frac{du}{\cancel{\sin(x)}}\right) = -\int (1 - u^2) \, du$$

$$= -\left(u - \frac{u^3}{3}\right) = -u + \frac{u^3}{3}$$

$$= -\cos(x) + \frac{\cos^3(x)}{3} + C$$

$$\int \ln(x) \, dx = \int 1 \ln(x) \, dx$$

$f(x) = \ln(x)$ $\quad g'(x) = 1$

$f'(x) = \frac{1}{x}$ $\quad g(x) = x$

$$= x \ln(x) - \int \frac{x}{x} dx$$

$$= x \ln(x) - \int 1 \, dx$$

$$= x \ln(x) - x + C$$